Math 2114 : Section 4.2: Introduction to Determinants

Definition: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the *determinant* of A is a scalar

$$\det(A) = |A| = ad - bc$$

Example 1: Calculate $\det(A)$ where $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$.

$|A| = 1(3) - (2)(5) = 3 - 10 = -7$

Definition: If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, then the *determinant* of A is a scalar

$$\det(A) = |A| = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Example 2: Calculate $\det(A)$ where $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

$|A| = 0 \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + 0 \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$

$= 0 + 0 - 1(1(6) - (4)(3)) = -1(-6) = 6$

Definition: If A is an $n \times n$ matrix where $n \geq 2$, then the *determinant* of A is a scalar

$$\det(A) = |A| = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det(\tilde{A}_{1j})$$

where $\tilde{A}_{1j}$ is the $(n-1) \times (n-1)$ matrix obtained by deleting row 1 and column j from the matrix A.

Example 3: Calculate $\det(A)$ where $A = \begin{bmatrix} 2 & 3 & 0 & 0 \\ 1 & 0 & -1 & 3 \\ 0 & 3 & -2 & 0 \\ 7 & 3 & -3 & 0 \end{bmatrix}$.

$$|A| = (-1)^{1+1} 2 \begin{vmatrix} 0 & -1 & 3 \\ 3 & -2 & 0 \\ 3 & -3 & 0 \end{vmatrix} + 3(-1)^{1+2} \begin{vmatrix} 1 & -1 & 3 \\ 0 & -2 & 0 \\ 7 & -3 & 0 \end{vmatrix} + \cancel{0(-1)^{1+3} \begin{vmatrix} 1 & 0 & 3 \\ 0 & 3 & 0 \\ 7 & 3 & 0 \end{vmatrix}} \to 0 + 0$$

$$= 2\left(0 - (-1)\begin{vmatrix} 3 & 0 \\ 3 & 0 \end{vmatrix} + 3\begin{vmatrix} 3 & -2 \\ 3 & -3 \end{vmatrix}\right) - 3\left(1\begin{vmatrix} -2 & 0 \\ -3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 0 \\ 7 & 0 \end{vmatrix} + 3\begin{vmatrix} 0 & -2 \\ 7 & -3 \end{vmatrix}\right)$$

$$= 2\left(-1(0-0) + 3(-9 - -6)\right) - 3\left(1(0) + 1(0 - 0) + 3(0 - -14)\right)$$

$$= 2 \cdot 3(-9 - (-6)) - (3 \cdot 3(0 - -14))$$

$$= 6(-3) - 9(14) = -18 - 126 = -144$$